MASS TRANSFER BETWEEN A FLAT-BOTTOM SIDE CAVITY AND FLOW IN A CONDUIT

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A procedure for calculating the concentration in the bottom zone of a flat-bottom side cavity of a conduit during the replacement of one gas by another is developed on the basis of experimental data. The procedure entails the determination of the volumetric flow rates between the gas in the cavity and the gas moving along the main conduit. It is shown that a significant intensification of mass transfer is observed in comparison with pure diffusion transfer for values of the dimensionless group $Re^{1.2}Sc^{0.5}(\ell/d)^{-0.3} > 10^4$.

Mass transfer between a gas situated in a flat-bottom side cavity of a conduit and a gas moving along the conduit itself is encountered in a number of engineering and technical problems. In [1-3] it is shown theoretically and experimentally that the two dimensional problem of fluid flow around a deep rectangular cavity is accompanied by the formation of a system of vortices, which decay with depth. It is obvious that the convective mass transfer between the flat-bottom cavity and the external flow is determined by the strength of these vortices and, hence, by the external flow velocity. Presser [4] has investigated convective mass transfer from three-dimensional cavities of various configurations and depths. The mass-transfer rate was determined from the change in weight of naphthalene and paradichlorobenzene deposited on the cavity wall during their sublimation into the flow. The results are displayed in the form of dimensionless groups: $Sh = f(Re, Sc)$, $10^2 \leqslant Re \leqslant 10^6$.

It was established that significant intensification of mass transfer between the flat-bottom cavity and the main flow is observed for $Re > 2 \cdot 10^3$. The mass-transfer coefficient increases at about half the rate for $Re > 10^4$. The investigations were carried out for flat-bottom cavities with depths of 0.5 to 3 times the diameter. However, the investigation was limited to a relatively shallow depth ($\ell/d = 0.5$ for cylindrical flat-bottom cavities) in the above-indicated range of rapid increase in mass transfer ($2 \cdot 10^3 \leq Re \leq 10^4$). For flat-bottom cavities with depths of 1, 2, and 3 times the diameter, mass-transfer data are given only for $Re > 10^4$.

Mass transfer between an airflow in a duct and naphthalene situated on the bottom of a cylindrical cavity with a depth of 0 to 2 times the diameter has been investigated [5]. The value of the number Re in relation to the cavity diameter was varied from 9030 to 88,300. The data of [4, 5] are in good agreement in the intersecting ranges of the parameters ($Re > 10^4$, $0.5 \leq \ell/d \leq 2$), despite certain differences in the experimental procedures (the naphthalene was sublimated from the entire surface of the cylinder in [4] and only from the bottom in [5]). This situation can be attributed to the fact that the "hydrodynamic" component, i.e., the volumetric flow rate between the cavity and the conduit, is a bottleneck that limits mass transfer.

Here we investigate the time variation of the concentration of a gas that initially fills up the entire flat-bottom cavity. In this perspective the statement of the problem differs considerably from [4, 5], but straightforward calculations can be used to reduce the results of the present study and those of [4, 5] to a unified relation between dimensionless groups.

We consider the problem for the case of indefinitely small gas velocities in the main conduit ($u \to 0$). We assume that the external flow does not introduce any perturbations into the region of the flat-bottom cavity in this case. These conditions reduce the mass transfer

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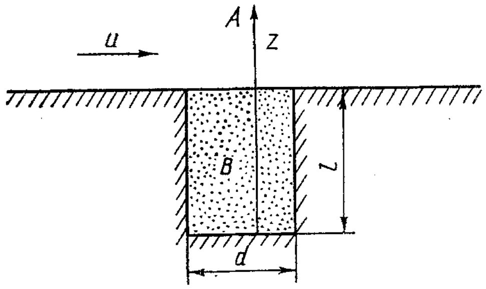

Fig. 1. Schematic diagram of the diffusion problem: A) external region filled with gas A; B0 flat-bottom cavity region filled with gas B; ℓ) depth of cavity; d) diameter of cavity; u) external flow velocity; z) coordinate axis.

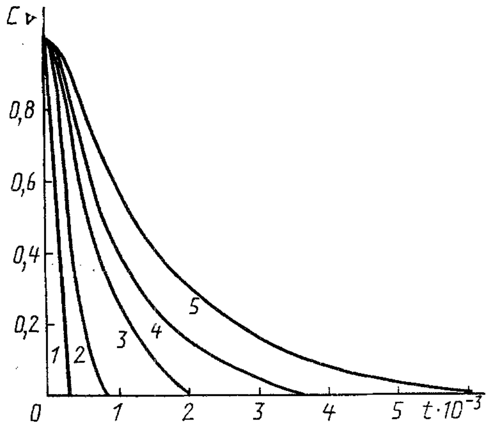

Fig. 2. Hydrogen concentration c_V (vol. fractions) vs time (sec) in the bottom zone of the flat-bottom cavity (z = 0). 1) u = 2 m/sec, ℓ = 0.3 m; 2) 1 m/sec, 0.3 m; 3) 0.27 m/sec, 0.3 m; 4) 1 m/sec, 0.5 m; 5) 0.27 m/sec, 0.5 m. For all the cavities d = 0.1 m.

to a pure diffusion process, for which an analytical solution exists under the following conditions (Fig. 1). At time t = 0 gas B occupies the cavity, and gas A exists outside the cavity. Then mass transfer sets in between the cavity and the external volume. We assume here that the concentration of gas B in the external volume is equal to zero at all times. This condition is physically equivalent to having the external space move with a certain small velocity to the left or to the right. The process is now described by the diffusion equation

$$\frac{\partial c}{\partial \tau}=\frac{\partial^2 c}{\partial \xi^2}. \tag{1}$$

The initial and boundary conditions have the form

$$\text{at } \tau=0 \text{ for } 0\leqslant\xi\leqslant 1 \quad c=1;$$
$$\text{at } \tau\geqslant 0 \text{ for } \xi\geqslant 1 \quad c=0.$$

Under these conditions Eq. (1) has a general solution in the form of an infinite series:

$$c=\sum_{n=0}^{\infty}\frac{4(-1)^n}{(2n+1)\pi}\exp\left[\frac{-(2n+1)^2\pi^2\tau}{4}\right]\cos\frac{(2n+1)\pi\xi}{2}. \tag{2}$$

Since the series on the right-hand side of Eq. (2) converges rapidly, it can be restricted to the one term corresponding to n = 0 with sufficient accuracy for practical calculations. Taking into account the condition c = 1 at τ = 0 and ξ = 0, we have

$$c\approx\exp\left(-\frac{\pi^2\tau}{4}\right)\cos\left(\frac{\pi}{2}\xi\right). \tag{3}$$

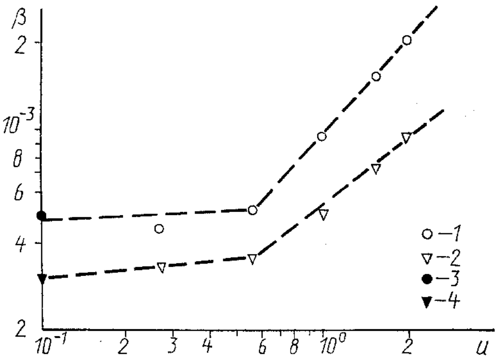

Fig. 3. Mass-transfer coefficient β (m/sec) vs velocity u (m/sec). 1) ℓ = 0.3 m; 2) ℓ = 0.5 m; 3, 4) theoretical values of β_0 at small velocities u: 3) cavity depth ℓ = 0.3 m; 4) 0.5 m.

It is evident from Eq. (3) that the maximum concentration at each instant occurs in the bottom zone (ξ = 0):

$$c_{\max} = \exp\left(-\frac{\pi^2\tau}{4}\right). \tag{4}$$

Consequently, Eq. (4) describes the variation of the concentration in the bottom zone of the cavity. Inasmuch as this concentration is a maximum, it serves as a suitable characteristic parameter of the process. We shall use only this characteristic concentration in the ensuing calculations. We shall also drop the subscript "max".

To study the influence of the flow velocity on the mass-transfer rate, we carried out experiments on the replacement of hydrogen by nitrogen in a cylindrical flat-bottom side cavity with an inside diameter of 100 mm and depths of 300 mm and 500 mm. The symmetry axis of the cavity was oriented perpendicular to the axis of the main conduit, which also had a diameter of 100 mm. The experimental procedure was as follows. After evacuation, the entire system was filled with hydrogen gas to a pressure just above atmospheric. The pressure was then equalized with the atmospheric pressure through a drainage line. Nitrogen gas began to flow from one end through the main conduit. The nitrogen—hydrogen mixture escaped into the atmosphere from the other end of the conduit. The temperatures of the hydrogen and the nitrogen were 18-20°C. The fast-reacting probe of a cooling-power gas analyzer [6] was built into the bottom of the flat-bottom cavity. The readings of the gas analyzer were plotted on a graphic recording instrument. The flat-bottom cavity was oriented with the bottom up in order to rule out the influence of buoyancy. The experiments yielded a family of curves of $c_V = f(t)$ (Fig. 2) for average nitrogen flow velocities in the conduit from 0.27 m/sec to 2 m/sec.

The mass transfer between the flat-bottom cavity and the main-flow conduit is conveniently described on the basis of the concept of volumetric flow rate of one of the components across a conditional interface between the flat-bottom cavity and the conduit. By definition,

$$j_B = -\frac{dV}{dt}\frac{1}{S}. \tag{5}$$

According to Fick's second law,

$$j_B = -j_A, \tag{6}$$

i.e., the displaced volume of hydrogen is replaced by an equal volume of nitrogen. There is sufficient basis to assume that the volumetric flow of one of the components across the cavity—conduit interface is proportional to the difference between the concentrations of this component in the conduit and in the cavity. For component B

$$j_B = \beta c. \tag{7}$$

It is evident from Fig. 2 that after a certain time essentially all of component B (hydrogen) with a volume

$$V_B = lS \tag{8}$$

is expelled from the cavity.

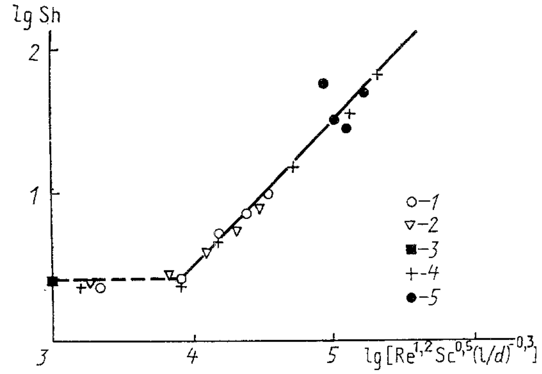

Fig. 4. Sherwood number vs Reynolds number. 1) ℓ = 0.3; 2) ℓ = 0.5 m; 3) theoretical Sherwood number Sh_0 for small Re; 4) data of [4] for ℓ/d = 0.5; 5) data of [5] for ℓ/d = 0.4-2.0; the solid line is calculated according to Eq. (17), and the dashed line is calculated according to Eq. (16).

According to Eq. (5),

$$V_B = \int_0^\infty j_B \, S dt. \tag{9}$$

Equating the right-hand sides of Eqs. (8) and (9) and taking Eq. (7) into account, we obtain the equation for the mass-transfer coefficient

$$\beta = \frac{l}{\int_0^\infty c dt}. \tag{10}$$

The coefficient β is determined from the experimental curves of the concentration in the bottom zone of the cavity (Fig. 2). For this purpose it is necessary to determine graphically the area under the c(t) curve and to calculate the coefficient β according to Eq. (10).

According to the analytical expression (4), the coefficient β_0 for pure diffusion mass transfer is

$$\beta_0 = \frac{l}{\int_0^\infty \exp\left(-\frac{\pi^2}{4}\frac{tD}{l^2}\right) dt} = \frac{\pi^2 D}{4l} \approx 2.5\,\frac{D}{l}. \tag{11}$$

For flow in the conduit with a sufficiently small velocity to establish the boundary conditions under which the solution (4) was obtained, the coefficient $\beta \to \beta_0$. This conclusion is well corroborated experimentally (Fig. 3).

These results can be extended to any pair of gases by dimensional analysis and similarity theory. The dimensionless mass-transfer coefficient $\beta^* = \beta/\beta_0$ is proportional to the Sherwood number Sh = βℓ/D. According to Eq. (11),

$$\mathrm{Sh} = 2.5\beta^*. \tag{12}$$

The general form of the functional dependence for the parameters characterizing diffusion—convection mass transfer between the flat-bottom cavity and the conduit is given by the relation

$$\beta = f(u, d, l, \nu, D). \tag{13}$$

On the basis of the Buckingham's π theorem [7] we reduce Eq. (13) to the form

$$\frac{\beta l}{D} = \mathrm{Sh} = f(\mathrm{Re}, \mathrm{Sc}, l/d). \tag{14}$$

According to Eq. (11), for small velocities u

$$\mathrm{Sh}_0 = \frac{\beta_0 l}{D} = 2.5. \tag{15}$$

This theoretical result is well corroborated by the experimental data (Fig. 4). It is evident from Fig. 4 that for a generalized argument $\lambda = Re^{1.2}Sc^{0.5}(\ell/d)^{-0.3} \leq 10^4$

$$Sh = Sh_0 = 2.5; \tag{16}$$

for $10^4 < \lambda \leq 2 \cdot 10^5$

$$Sh = 3.16 \cdot 10^{-4}\lambda. \tag{17}$$

Equations (16) and (17) can be used solve two different problems of mass transfer between a flat-bottom cavity and an external flow. The first problem is the expulsion of the sublimated component from the cavity. Its solution requires the substitution of the dimensionless parameters in Eqs. (16) and (17). The second problem is the variation of the concentration in the cavity as one gas is replaced by the other. This problem is solved as follows.

The volume-average concentration in the flat-bottom cavity is approximately equal to c/2. The volume occupied by component B at a given time is

$$V \approx \frac{1}{2} Slc.$$

According to this expression,

$$dV = \frac{1}{2} Sldc. \tag{18}$$

The simultaneous solution of Eqs. (5), (7), and (18) yields the relation

$$t = \frac{1}{2} \frac{l}{\beta} \ln \frac{c_0}{c}, \tag{19}$$

or

$$t = \frac{1}{2} \frac{l^2}{Sh\, D} \ln \frac{c_0}{c}. \tag{20}$$

We round out the foregoing discussion with some general remarks. Mass transfer between a flat-bottom cavity and an external flow does not depend on the geometry of the external duct. Appreciable intensification of mass transfer is observed for values of the dimensionless group $\lambda = Re^{1.2}Sc^{0.5}(\ell/d)^{-0.3} > 10^4$. For values of this group higher than $2 \cdot 10^5$ the mass-transfer coefficient increases at about half the rate [4, 5], so that mass-transfer processes are essentially useful for $10^4 < \lambda < 2 \cdot 10^5$.

NOTATION

Re, Reynolds, number; $Sh = \beta\ell/D$, Sherwood number; Sc, Schmidt number; ℓ, depth of flat-bottom cavity; d, inside diameter of flat-bottom cavity; u, average flow velocity of gas in conduit; c, volume concentration; β, mass-transfer coefficient; D, diffusion coefficient; $\tau = tD/\ell^2$, dimensionless time; $\xi = z/\ell$, dimensionless coordinate; V, volume of flat-bottom cavity; S, cross section of flat-bottom cavity; j_B, volumetric flow rate of gas B (hydrogen).

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